High-density thick-beam free-electron laser

Amnon Fruchtman
Department of Nuclear Physics, Weizmann Institute of Science, 76100 Rehovot, Israel
(Received 16 September 1987)

A free-electron laser, which employs a high-density thick electron beam, of a thickness comparable to the wiggler wavelength, is considered. Full transverse dependence of the wiggler field as well as equilibrium self-fields of the beam are taken into account. We identify a domain in parameter space where the electron beam is wholly resonant and the gain scales as in the one-dimensional theory of the free-electron laser in the Raman regime. A family of electron-density radial profiles is shown to have a low shear in the resonance parameter and thus to exhibit a particularly high gain.

I. INTRODUCTION

In order to increase the gain and the power in the free-electron laser (FEL),[1] it is desirable to employ a high-current electron beam. Practically, because of the strong repulsive self-fields of the beam, it is difficult to generate a high-current electron beam with a small cross section. Therefore, an increase in the current of the electron beam is usually accompanied by an increase in the cross section. However, the increase of the electron beam cross section in FEL's is usually limited by the requirement that the radial dimensions of the beam be much smaller than the wiggler wavelength. Usually, when the beam cross section becomes comparable to the wiggler wavelength, the radial gradients of the wiggler field and of the equilibrium self-fields of the beam introduce shear into the parameters determining the resonance conditions, and prevent the beam from being wholly resonant. Thus, FEL experiments which employ high-current electron beams with large cross sections are usually forced to employ wigglers with long wavelengths. For an electron beam of a given energy, the increase in the wiggler wavelength is followed by an increase in the radiation wavelength. One of the consequences of a large cross section of the beam and the resulting long wavelength of the wiggler is therefore, the difficulty in generating short-wavelength radiation. For example, the Livermore FEL, which employed an 850-A electron beam of a 0.8-cm radius together with a wiggler of a 9.8-cm wavelength, produced radiation of an 8.6-mm wavelength.[2] Nevertheless, by performing a stability analysis of an equilibrium which was derived by a new formalism,[3] we have recently shown[4] that under certain conditions, a low-density thick electron beam, of a thickness comparable to the wiggler wavelength, can be wholly resonant. A FEL employing such a beam would operate in the strong-pump regime. In this paper we explore the possibility of employing a high-density thick electron beam in a FEL operating in the Raman regime with an even higher gain. We show that there is a family of radial density profiles which have small shear in the resonance parameter and thus exhibit a particularly high gain. We identify also constraints on the parameters in the thick-beam FEL which limit its advantages. This study of a high-density thick beam FEL differs from previous three-dimensional analytical theories which dealt with beams of transverse dimensions smaller than the wiggler wavelength and which neglected equilibrium self-fields.[5-7]

The possibility of employing high-density thick electron beams has significant practical importance. For a given density, one could increase the current by increasing the beam cross section without increasing the wiggler wavelength. On the other hand, one could decrease the wiggler wavelength in order to obtain radiation of shorter wavelength without necessarily having to decrease the beam cross section and total current. Thus the use of high-density thick electron beam enables one to increase the total current and to improve the gain and power of FEL's for various radiation wavelengths.

In Sec. II we derive the governing equation. In Sec. III we show that the shear in the resonance parameter is expressed through the presence of a continuous spectrum in the eigenvalue equation. The vanishing of the shear for a specific family of radial density profiles thus corresponds to the shrinking of the continuous spectrum to a point and to an increase in the gain. We demonstrate this by numerical examples.

II. DERIVATION OF THE GOVERNING EQUATION

A. The equilibrium

We consider a helical cold flow of electrons, which, in its steady state, depends only on r and ϕ ($=\theta-kz$), and which is driven by an M multiple external magnetic helical wiggler and is confined by a uniform axial magnetic field. For the equilibrium we use the formalism developed by Weitzner *et al.*[3] for a relativistic helically symmetric cold fluid, and expand the quantities, as in Refs. 3 and 4, in the small parameter ϵ. This parameter measures the ratio of the magnitudes of the perpendicular and parallel momentum components, as well as the ratio of the magnitudes of the external wiggler and uniform axial magnetic fields. Contrary to Ref. 4, where we studied the low-density case of density second order in ϵ, here we allow the density to be first order in ϵ. In this case of high density, the equilibrium self-fields appear in the relevant low-order expansion and play a major role in the stability analysis. We assume also that to lowest order the beam is monoenergetic, with kinetic energy $(\gamma-1)mc^2$, where

$$\gamma=O(\epsilon^{-p_\gamma}),\quad p_\gamma\geq 0$$

and that to lowest order the beam has no perpendicular momentum. The approximated equilibrium under these assumptions was presented in Ref. 3. In this subsection we state the main results.

The normalized magnetic field $\underline{B}$ [the magnetic field multiplied by $e/(mc^2k)$, where e and m are the electron charge and mass, and c is the velocity of light in vacuum] is of the form

$$B_r=B_r^{(+M)}(r)e^{iM\phi}+B_r^{(-M)}(r)e^{-iM\phi}+O(\epsilon^{2-p_\gamma}),$$

$$B_\theta=B_\theta^{(+M)}(r)e^{iM\phi}+B_\theta^{(-M)}(r)e^{-iM\phi}+B_\theta^{(0)}(r)+O(\epsilon^{2-p_\gamma}),$$

$$B_z=B_0+B_z^{(+M)}(r)e^{iM\phi}+B_z^{(-M)}(r)e^{-iM\phi}+O(\epsilon^{2-p_\gamma}).$$

The explicit form of the various coefficients can be extracted from Ref. 3. Here $B_r^{(\pm M)}$, $B_\theta^{(\pm M)}$ and $B_z^{(\pm M)}$ are the wiggler field components and $B_\theta^{(0)}$ is the azimuthal self-field of the beam. They are all $O(\epsilon^{1-p_\gamma})$. The axial component of the wiggler field is comparable both to the transverse components of the wiggler field and to the azimuthal self-magnetic field of the beam. The external uniform field B_0 is $O(\epsilon^{-p_\gamma})$. The normalized density h [the density multiplied by $4\pi e^2/(mc^2\gamma k^2)$] and the normalized electrostatic potential [the potential multiplied by $e/(mc^2)$] are given as

$$h=h_1(r)+O(\epsilon^2),$$

$$\Phi=\Phi_1(r)+O(\epsilon^{2-p_\gamma}),$$

where h_1 is $O(\epsilon)$, Φ_1 is $O(\epsilon^{1-p_\gamma})$, and they are related by Gauss's law,

$$(r\Phi_{1,r})_{,r}/(k^2r)=h_1\gamma_0\,. \tag{1}$$

The beam energy is

$$\gamma=\gamma_0+\gamma_1(r)+O(\epsilon^{2-p_\gamma}),$$

where γ_1 is $O(\epsilon^{1-p_\gamma})$. Also, γ and Φ are related through

$$\gamma-\Phi=E(\chi)\,.$$

Here E is the total energy of the fluid and χ is the stream function,

$$\chi=B_0+\chi_1(r)+O(\epsilon^{2-p_\gamma}),$$

where χ_1 is $O(\epsilon^{1-p_\gamma})$. In the case of low density, Φ_1 was zero and we chose an equilibrium of a particular energy profile $\gamma_1(r)$.[4] In the present analysis, we treat the more physical case of uniform total energy, $E(\chi)=\gamma_0$, and thus

$$\gamma_1-\Phi_1=0\,. \tag{2}$$

Both γ_1 and Φ_1 are functions of r only. The cylindrical components of the normalized momentum (the momentum divided by mc) are

$$u=\frac{MA/(kr)\{\alpha r[I_M(Mkr)]_{,r}+M^2I_M(Mkr)\}\cos(M\phi)}{(M^2-\alpha^2)}+O(\epsilon^{2-p_\gamma}),$$

$$v=\frac{-M^2A/(kr)\{r[I_M(Mkr)]_{,r}+\alpha I_M(Mkr)\}\sin(M\phi)}{(M^2-\alpha^2)}+\frac{\chi_1(r)}{(krB_0w_0)}+O(\epsilon^{2-p_\gamma}),$$

$$w=w_0+(\gamma_0/w_0)\gamma_1+O(\epsilon^{2-p_\gamma}),$$

and A is $O(\epsilon^{1-p_\gamma})$. The term proportional to χ_1 in v expresses the azimuthal drift of the beam due to the external axial magnetic field and the self-magnetic and electrostatic fields. Here α is B_0/w_0. Some of the quantities from Ref. 3 have been made nondimensional. The normalized density here is the normalized density of Ref. 3 divided by k^2. The magnetic field and the stream function are divided here by k.

In the present analysis, the axial component of the wiggler field does not contribute to the momenta to lowest order. The axial component of the wiggler field is comparable to the transverse components, but the transverse velocity is smaller than the axial velocity. As a result, the force which is a product of the axial wiggler field and the transverse velocity is smaller than the force which is a product of the transverse wiggler field and the axial velocity. However, near the resonance between the wiggler and the guide fields, the contribution of the axial component of the wiggler field could be crucial,[6] in which case our expansion would be invalid. In order to exclude the above resonance, we require that $M^2-\alpha^2$ is not too small and explicitly that

$$M^2-\alpha^2=O(1)\,.$$

B. The linearized equations

We now turn to the stability analysis. We seek a solution for the perturbed quantities of the form

$$\delta f=\left[\sum_{l=-\infty}^{\infty}\delta f^{(l)}(r)\exp(il\phi)\right]\exp[i(qz-\omega ct)]\ . \quad (3)$$

We limit ourselves to the case of the FEL resonance, $p_\omega=2p_\gamma$, where p_ω expresses the order of magnitude of ω/k,

$$\omega/k=O(\epsilon^{-p_\omega})\ .$$

We also assume the eigenvalue q to be such that q/k is also $O(\epsilon^{-p_\omega})$. We substitute the form (3) for the perturbed quantities in the linearized cold fluid and Maxwell's equations, and obtain an infinite set of coupled ordinary differential equations. Careful examination of the orders of magnitude of the various quantities shows that the momentum equation for $\delta w^{(l-M)}$, the continuity equation for $\delta h^{(l-M)}$, and Gauss's law for $\delta E_z^{(l-M)}$ decouple from the rest of the equations under the following three conditions: if

$$p_\omega=1\ ,$$

if for some l, the normalized eigenvalue ν,

$$\nu\equiv\frac{1}{w_0}\left[-\frac{\omega}{k}\gamma_0+\left[\frac{q}{k}-l+M\right]w_0\right]\ ,$$

is $O(\epsilon)$, and if the normalized wave electric field $\delta\underline{E}$ [the field multiplied by $e/(mc^2k)$] has only l component to lowest order. A similar truncation procedure was performed in Ref. 4 for the low-density case. Here, because the density is higher, we have to use Gauss's law and cannot neglect $\delta E_z^{(l-M)}$. This corresponds to an operation in the Raman regime. Solving this finite set of algebraic equations gives us the perturbed density

$$\delta h^{(l-M)}=-\frac{ih_1(\omega/k)(u^{(-M)}\delta E_r^{(l)}+v^{(-M)}\delta E_\theta^{(l)})}{w_0^4[\nu-g_+(r)][\nu-g_-(r)]}\ ,$$

where

$$g_\pm(r)=-\frac{\omega}{kw_0^3}\Phi_1+\frac{\omega}{kw_0\gamma_0}(\,|u_1^{(M)}|^2+|v_1^{(M)}|^2)\pm\frac{h^{1/2}}{w_0}\ .$$

We used the equality (2) and neglected the drift velocity $\chi_1/(krB_0\omega_0)$, since in our case of a relativistic beam ($p_\gamma=\frac{1}{2}$), this velocity is $O(\epsilon^{3/2})$. The quantities $u_1^{(\pm M)}$ ($\equiv\int_0^{2\pi}d\phi\, e^{\mp iM\phi}u$) are $O(\epsilon^{1/2})$. The functions $g_\pm(r)$ express the shear in the resonant denominator. They consist of three terms; the first represents the influence of the self-electrostatic field, the second represents that of the radial dependence of the wiggler field, and the third represents that of the radial dependence of the beam density. However, in our ordering, they are all $O(\epsilon)$ as is the eigenvalue ν. A different choice of the parameters will usually result in a ν that is smaller than the other terms, and thus weaken the instability. Because of the particular ordering of the various quantities, the expansion is uniform across the beam. The whole beam is resonant and the perturbed density is large and is of the same order of magnitude across the beam. We now substitute the perturbed current into Maxwell's equations and obtain the final set of equations,

$$\frac{1}{k}\left[r\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\right]+\xi-2\nu\omega k-\frac{l}{r^2}(l\pm2)\right](r\delta E_r^{(l)}\pm ir\delta E_\theta^{(l)})=\frac{h_1(\omega/k)^2(u^{(M)}\pm iv^{(M)})(u^{(-M)}\delta E_r^{(l)}+v^{(-M)}\delta E_\theta^{(l)})}{w_0^4(\nu-g_+)(\nu-g_-)}\ . \quad (4)$$

The boundary conditions are the regularity of $\delta E_r^{(l)}$ and $\delta E_\theta^{(l)}$ at the origin, and

$$\frac{\partial}{\partial r}(r\delta E_r^{(l)})_{r=R}=0=\delta E_\theta^{(l)}(R)\ ,$$

following the assumption that a perfectly conducting wall is located at $r=R$. The mismatch parameter ξ is

$$\left[\omega^2-\left[\omega\frac{\gamma_0}{w_0}-kM\right]^2\right]$$

and ξ/k^2 is $O(1)$. The right-hand sides of Eqs. (4) are $O(1)$, and thus to lowest order we do not have the well-known vacuum waveguide modes, nor can we use perturbation techniques to find the eigenvalues. These have to be found by a numerical solution of the differential equations (4). The eigenvalue ν is $O(\epsilon)$ and not $O(\epsilon^{4/3})$ as in Ref. 4. Thus the gain here is expected to be higher. The eigenvalue ν is

$$O[(h\omega/k)^{1/4}u/w]\ ,$$

similar to the scaling in the one-dimensional (1D) theory of the FEL in the Raman regime. Thus we have found a domain in parameter space where the shear does not distort the scaling.

A thick-beam configuration which scales as in the one-dimensional theory is important in the following sense. For an electron beam of given energy, perpendicular velocity and density, the actual gain in a FEL is usually smaller than the gain predicted by the 1D theory. This is, firstly, because the gain attenuates according to the filling factor, which reflects the fact that the electron beam cross section is smaller than the wave beam cross section. Secondly, this is because not all the electron beam is resonant, as a result of inhomogeneities introduced by the transverse wiggler gradients and the static self-fields of the beam. One could increase the filling factor by positioning the beam in such a way as to overlap the peak of the wave mode or by exploiting a self-focusing of the wave. If possible, one could also increase the density by reducing the cross section of the beam. In the latter case, the gain may increase because of the density increase, despite the increase in diffraction which reduces the gain. However, for given beam energy, perpendicular velocity, and density, the use of a thick electron beam which is wholly resonant with a cross section similar to the cross section of the wave, as we suggest, provides a natural way of increasing the filling factor to order 1. In this sense, the scaling of the gain as in the 1D model corresponds to a higher gain. It is true that, as was shown here, operating the thick-beam FEL is subject to several constraints. Particular relations between the orders of magnitude of the various quantities have to be satisfied. For example, if the frequency were increased, both the density and the transverse velocity would have to be decreased, and this would result in a reduction of the gain. Thus these constraints limit the advantages of the thick-beam FEL. One may prefer a lower-current thin beam, where the accompanied diffraction of the radiation reduces the gain, but where there is a larger wiggler field than the one allowed in the thick-beam configuration. Which configuration results in a higher gain depends on the particular parameters under consideration. For cases where there are bounds on the wiggler intensity and on the beam density, the thick-beam FEL configuration suggests an additional option for increasing the gain by increasing the cross section of the beam and the total current.

One should note which are the equilibrium quantities that affect the final form of the linearized equations (4). The equilibrium transverse momentum appears in both the numerator and the resonance denominator of the source term on the right-hand side (rhs) of the equation. In deriving the expression for the transverse momentum the full transverse dependence of the wiggler field was taken into account. The equilibrium self-fields as well as the axial wiggler component affect the equilibrium transverse momentum to higher order only. However, the equilibrium self-fields do affect Eq. (4) in a crucial way through their presence in the resonance denominator, determining the radial dependence of γ [Eq. (2)]. Thus both the full transverse dependence of the wiggler field and the equilibrium self-fields appear in the governing equation (4).

III. REDUCING THE SHEAR BY TAILORING THE DENSITY RADIAL PROFILE

In this section we examine the conditions under which the shear in the resonance parameter is reduced and the gain is increased. The eigenvalues ν which satisfy, for some r between zero and R, the equalities

$$\nu=g_{+}(r) \tag{5a}$$

or

$$\nu=g_{-}(r)\ , \tag{5b}$$

comprise the continuous parts of the spectrum. The nonreal eigenvalues are located in the neighborhood of the continuous spectrum, which is real. It is plausible that when the continuous spectrum becomes larger, the imaginary parts of the nonreal eigenvalues become small. This can be explained physically, since a large continuous spectrum corresponds to a large shear in the resonance parameter, which results in the thinning of the resonant layer. The unstable mode becomes localized near the thin resonant layer, and the growth rate decreases. Thus, in order to increase the gain, conditions should be sought under which the continuous spectrum shrinks.

We look for density profiles for which g_{+} or g_{-} is a constant. By using Eq. (1), we obtain the following nonlinear equation for Φ_1:

$$r\Phi_{1,rr}+\Phi_{1,r}-r\gamma_0\left[\frac{\omega}{w_0^2}\Phi_1-\frac{\omega}{2\gamma_0}(\,|u_1^{(M)}|^2+|v_1^{(M)}|^2)-\beta\right]^2=0\ , \tag{6}$$

where Φ_1 and its derivative should be regular. We checked the assumption that the growth rate is high for these density profiles as follows. We picked a constant β and solved Eq. (6) for the density profile $h_\beta(r)$. Then we solved Eqs. (4) for various ξ's and looked for the highest imaginary part of ν. We varied ξ again, and looked for the highest imaginary part of ν for other density profiles which are related to the preferred density profile $h_\beta(r)$ through

$$h_\beta^{(s)}(r)\equiv sh_\beta(r)\ . \tag{7}$$

By varying the parameter s, we vary the total current of the beam while keeping the relative density profile unchanged. When s equals 1, the current is that of the preferred density profile.

In Fig. 1 three preferred normalized density profiles $\omega h_1/k$ are plotted for three values of the parameter β. In Fig. 2 the maximum normalized growth rate $(\omega/k)\mathrm{Im}\nu$ for the density profiles $h_\beta^{(s)}$ is plotted versus s for these three values of β. When s is increased towards the value 1, the growth rate increases rapidly. This is a combined effect of the increase of the density and of the shrinking of the shear. When s is increased above the value 1, the increase in the growth rate stops and the growth rate soon *decreases* despite the *increase* in the density. For s larger than 1, the increase in the shear dominates. In the numerical examples, kR is 1.5, $l=1=M$, and the normalized wiggler strength $A^2\omega/k$ is 1. The unstable mode was that of one node, which for the vacuum case is reduced to TE_{11}.

We now give numerical examples. As a first example, let us assume that ω/k equals 50. For $k=2.25\ \mathrm{cm}^{-1}$, the radiation wavelength is approximately 550 μm and the waveguide radius is 0.7 cm. The wiggler and the guide field intensities are 545 G and 11 kG, respectively. The total beam current is 3.8 kA for the preferred density profile ($\beta=-1.5$). The beam energy is 2.04 MeV. The growth rate is 0.043 cm^{-1} (39 dB/m). As a second example, we assume that ω/k equals 10 and $k=1.5\ \mathrm{cm}^{-1}$. Now the radiation wavelength is larger, approximately 4.1 mm, and the waveguide radius is 1.05 cm. The wiggler and the guide field intensities are 817 G and 3.2 kG, respectively. The total beam current is again 3.8 kA for the same preferred density profile. The beam energy is 610 keV. The growth rate is 0.142 cm^{-1} (129 dB/m).

The high gain in these examples was achieved by having a high-current thick beam which is wholly resonant.

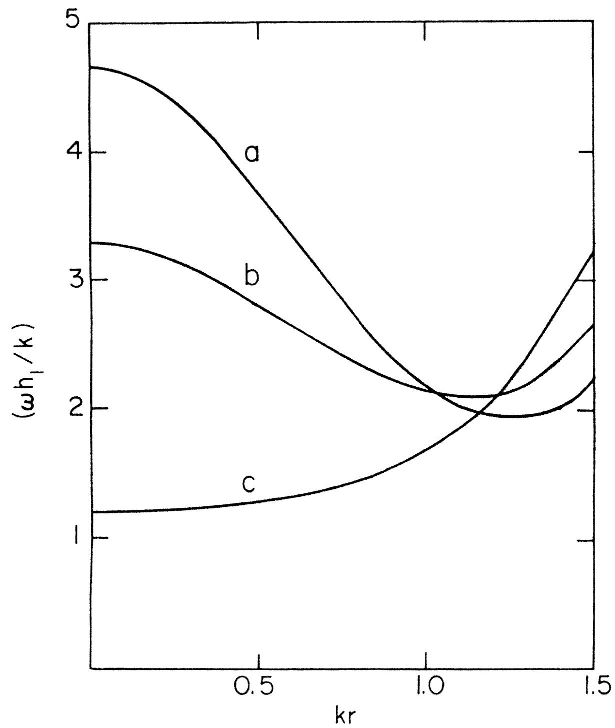

FIG. 1. The preferred normalized density profiles [$s=1$ in Eq. (6)] for (a) $\beta=-1.5$, (b) $\beta=-1.0$, and (c) $\beta=0$.

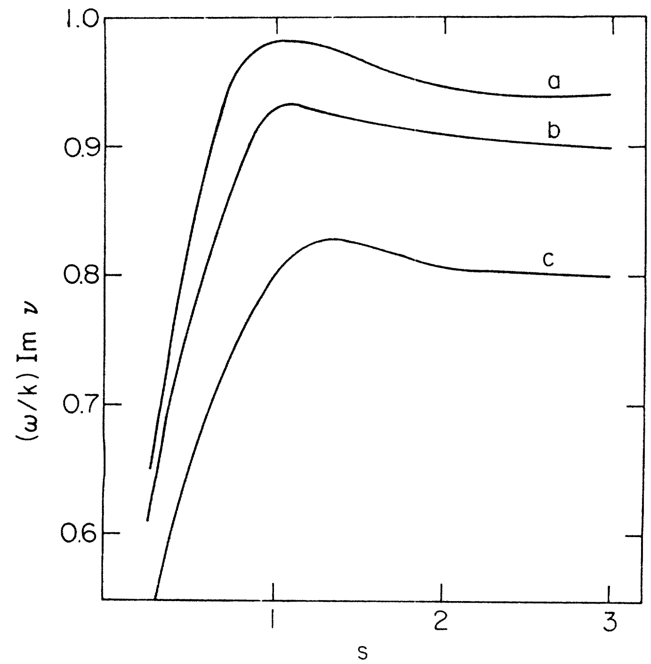

FIG. 2. The maximum normalized growth rate vs s for (a) $\beta=-1.5$, (b) $\beta=-1.0$, and (c) $\beta=0$.

As mentioned before, there is an accompanied constraint on the magnitude of the transverse velocity. While we explored here a way to increase the gain by increasing the current of the beam, there is an alternative way of increasing the gain by propagating a lower-current thin beam but with a larger wiggler field. The thick-beam FEL is not always preferable to such alternative ways, but it is an additional option, not dealt with before, which in some cases may be advantageous.

We note also that all the cases which satisfy the conditions we described in order for the high-density beam to be wholly resonant have input powers of the same order of magnitude. The current is proportional to $h\gamma k^2$ and thus the power is proportional to $h\omega k$, which is $O(k^2)$. Because of the high currents, even with a modest efficiency and without wiggler tapering, the output power of the thick-beam FEL will be high. Since the whole thick beam is resonant, it is reasonable to assume that the efficiency will not be lower than in the usual thin-beam FEL. An efficiency of 6.7% will result will result in an output power of 0.5 GW in the above first example and of 160 MW in the second example.

One could argue that beam radial-profile control is not easy to achieve. However, the results are not very sensitive to deviations from the preferred density profiles. Once the flow parameters of a thick-beam FEL have been chosen to satisfy the relations described above, it is sufficient to employ a beam of a density profile similar to the preferred profile in order to increase the gain.

In summary, in this paper the possibility of a high-density thick-beam FEL was explored. We studied the FEL interaction with full transverse dependence of the wiggler field and with equilibrium self-fields of the beam. We identified a domain in parameter space, where the gain scales as in the 1D theory of the Raman regime, despite the shear introduced by the radial gradients. The shear in the resonance parameter was expressed as a continuous spectrum of eigenvalues. It was shown that by tailoring the electron-density radial profile, one could eliminate this shear and thus increase the gain.

ACKNOWLEDGMENTS

The author is grateful to Dr. I. Ben-Zvi and to Mr. E. Jerby for helpful discussions. The author acknowledges financial support from the Sir Charles Clore Foundation.

[1]T. C. Marshall, *Free Electron Lasers* (Macmillan, New York, 1985), and references therein.

[2]T. J. Orzechowski, B. R. Anderson, J. C. Clark, W. M. Fawley, A. C. Paul, D. Prosnitz, E. T. Scharlemann, S. M. Yarema, D. B. Hopkins, A. M. Sessler, and J. S. Wurtele, Phys. Rev. Lett. **57**, 2172 (1986); T. J. Orzechowski, E. T. Scharlemann, B. Anderson, V. K. Neil, W. M. Fawley, D. Prosnitz, S. M. Yarema, D. B. Hopkins, A. C. Paul, A. M. Sessler, and J. S. Wurtele, IEEE J. Quantum Electron. **QE-21**, 831 (1985).

[3]H. Weitzner, A. Fruchtman, and P. Amendt, Phys. Fluids **30**,

539 (1987).
[4]A. Fruchtman, Phys. Fluids **30**, 2496 (1987).
[5]H. S. Uhm and R. C. Davidson, Phys. Fluids **24**, 1541 (1981).
[6]H. P. Freund and A. K. Ganguly, Phys. Rev. A **28**, 3438 (1983).
[7]B. Steinberg, A. Gover, and S. Ruschin, Phys. Rev. A **36**, 147 (1987).